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Screening Examination Questions・Answer Sheet

Subject	Mathematics

Score

【 Q1 】 Find the derivative of the following functions.（ 16 points ）

(1) $y = x\log_e x - x$ (e: base of natural logarithm)

Answer $\dfrac{dy}{dx} = \log_e x$

(2) $y = x^{\sin x}$

Answer $\dfrac{dy}{dx} = x^{\sin x}\left(\cos x \log_e x + \dfrac{\sin x}{x}\right)$

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Screening Examination Questions・Answer Sheet

Subject	Mathematics

Score

【 Q2 】 Solve the following questions.（ 16 points ）

(1) $\int_0^{\frac{\pi}{2}} \sin\theta\cos\theta \, d\theta$

Answer $\frac{1}{2}$

(2) $\iint_D xy \, dxdy \qquad D = \left\{x^2 + y^2 \le a^2, x \ge 0, y \ge 0\right\}$ (a is a positive constant)

Answer $\frac{a^4}{8}$

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【 Q3 】 Solve the differential equations under the given initial condition.（ 20 points ）

(1) $\frac{dy}{dx}+ay=-b \quad (I.C. \quad x=0, y=0)$ (a and b are positive constants)

Answer $y=-\frac{b}{a}+\frac{b}{a}e^{-ax}$

(2) $\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=4 \quad \left(I.C. \quad x=0, y=4 \; and \; \frac{dy}{dx}=3\right)$

Answer $y=e^x+e^{2x}+2$

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Score

【 Q4 】 Solve the following questions.（ 16 points ）

(1) Find the area of the triangle with vertices A, B, and C.

$A(1,0,7)$, $B(2,1,8)$, $C(1,0,3)$

Answer $2\sqrt{2}$

(2) Find the volume of the tetrahedron with vertices A, B, C, and D.

$A(1,0,7)$, $B(2,1,8)$, $C(1,0,3)$, $D(2,2,9)$

Answer $\frac{2}{3}$

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Screening Examination Questions・Answer Sheet

Subject	Mathematics

Score

【 Q5 】 Answer the following questions about matrix $\mathbf{A}$. $\mathbf{A}=\begin{pmatrix}2 & 1\\ 1 & 2\end{pmatrix}$ （ 16 points ）

(1) Find the eigenvalues and eigenvectors of matrix $\mathbf{A}$.

Answer

Eigenvalue $\lambda=1$ then, Eigenvector $k\begin{pmatrix}1\\ -1\end{pmatrix}$ k is any nonzero constant.

Eigenvalue $\lambda=3$ then, Eigenvector $k\begin{pmatrix}1\\ 1\end{pmatrix}$ k is any nonzero constant.

(2) Diagonalize matrix $\mathbf{A}$.

Answer $\begin{pmatrix}3 & 0\\ 0 & 1\end{pmatrix}$ or $\begin{pmatrix}1 & 0\\ 0 & 3\end{pmatrix}$

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Screening Examination Questions・Answer Sheet

Subject	Mathematics

Score

【 Q6 】Let the probability density function of random variable X be given as follows.（ 16 points ）

$$f(x)=\begin{cases}\frac{3}{4}\left(1-x^2\right) & (-1\le x\le 1)\\ 0 & (x<-1, 1<x)\end{cases}$$

(1) Find the expected value of X.

Answer $E(X)=0$

(2) Find the variance of X.

Answer $V(X)=\frac{1}{5}$